



# UNIVERSITY OF CAMBRIDGE

## PART III SEMINAR SERIES

**MICHAELMAS 2018**

## ABSTRACTS

Thursday November 29 2018

Friday November 30 2018

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*Part III Seminar Series Directors*

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# 1 Analysis

## 1.1 Linden Disney-Hogg: Proof of the Arnol'd Liouville Theorem

Friday, November 30, 15:10 - 15:50 (MR5)

*Chair:* Kasi Wyczesany

Integrability allows us to find exact non-perturbative solutions to the system we are investigating. The Arnold-Liouville theorem from Hamiltonian dynamics gives us sufficient conditions for such integrability, and constructs the solution, in an elegant manner. I will hopefully provide some background and history, before presenting the proof of the theorem.

## 1.2 Tim Graute: Uniqueness of entropy

Friday, November 30, 15:50 - 16:30 (MR5)

*Chair:* Kasi Wyczesany

The Khinchin theorem states that, given three conditions, among them maximality of entropy for equidistributed sets, additivity of conditional entropy and invariance of entropy against the inclusion of 0 into the multiset, entropy as a continuous function from a multiset of non-negative numbers adding up to 1 into the real numbers is determined up to scaling. Khinchin's clearly and elementarily written proof for this statement shall be presented.

# 2 Probability and Statistics

## 2.1 Harvey Klyne: The Dantzig selector

Friday, November 30, 15:10 - 15:50 (MR4)

*Chair:* Marin Ballu

The Dantzig Selector is a method for high dimensional sparse regression. The form will be immediately reminiscent of the Lasso, however can suggest somewhat different models.

## 2.2 Zheneng Xie: Electrical networks in probability

Friday, November 30, 16:40 - 17:20 (MR5)

*Chair:* Piet Lammers

Many problems in probability are easy to state but difficult to prove. As an example, consider a finite connected graph. Such a graph will have finitely many spanning trees so pick one uniformly at random, this random object is called the uniform spanning tree (UST). Let  $e$  and  $f$  be distinct edges. Suppose I tell you  $e$  is in the UST, does this decrease

or increase the probability that  $f$  is in the tree? Surprisingly to answer this question we need to use ideas from electrical networks, like those you've met in secondary school.

### **2.3 Mathias Hockmann: Option pricing using importance sampling**

Friday, November 30, 17:20 - 18:00 (MR5)

*Chair:* Piet Lammers

Options are financial products traded at stock exchange markets and they give the buyer the chance to carry out a transaction at a future point and at predetermined conditions. What are "fair" prices for such options? How can those prices be estimated? How can we improve such estimates? I'll introduce the Black Scholes modell as a basic financial model and apply the proposed price estimation technique to a simple numerical example. No prerequisites expected!

## **3 General Relativity**

### **3.1 Jonathan Crabbe: Space, a final frontier?**

Thursday, November 29, 13:40 - 14:20 (MR2)

*Chair:* Omar Darwish

Through this talk, the different possibilities for the topology and the geometry of a FLRW universe according to the data of Planck 2015 mission are investigated

### **3.2 Ian Lim: A survey of singularities**

Thursday, November 29, 14:20 - 15:00 (MR2)

*Chair:* Omar Darwish

In the study of classical gravity, singularities often emerge when one tries to describe even the simplest spacetimes. However, it also seems to be true that in many cases, these singular points or surfaces are somehow hidden away from physical observers, e.g. by event horizons or intrinsic instabilities. In this talk, I will describe some of the main theorems regarding singularities in general relativity (due mainly to Penrose and Hawking) and discuss some of the difficulties in precisely defining what a singularity is. I will also introduce two conjectures regarding so-called "cosmic censorship" and outline some recent work on a proposed violation of the strong cosmic censorship conjecture.

### **3.3 Filipe Miguel & Francisco Duque: Motion in time-periodic backgrounds with applications to ultralight dark matter haloes at galactic centers**

Thursday, November 29, 15:10 - 15:50 (MR2)

*Chair:* Owain Salter Fitz-Gibbon

Historically, the motion of test particles has been used to test general relativity. In recent work (DOI: 10.1103/PhysRevD.98.024037), we showed how motion in spherically symmetric but time-dependent periodic backgrounds can lead to orbital resonances, even when dissipative effects are considered.

These specific geometries are known to describe a class of compact object - scalar oscillations. They also arise in the context of ultralight dark matter, where solutions to the field equations are interpreted as galactic haloes. We applied our previous developments to both these systems. Remarkably, our results indicate that the motion of S-2 like stars may carry distinguishable observational imprints of ultra-light dark matter.

### **3.4 Guillaume Dideron: Gravitational waves in Jordan-Brans-Dicke theory**

Thursday, November 29, 15:50 - 16:30 (MR2)

*Chair:* Owain Salter Fitz-Gibbon

We will review the simplest possible modification to GR: the addition of a scalar field. We will first discuss the dynamics of this modified theory, before deriving how its gravitational waves differ from the ones in GR. We will find that radiation is generated even in spherically symmetric problems, which does not happen in GR. The detection of a gravitational signal from a spherically symmetric system would thus mark a clear departure from GR. In spite of their symmetry, these systems are not easy to solve. We will discuss a general set-up, then explain some numerical results.

### **3.5 Carmen Jorge-Diaz: Applications of the AdS/CFT in condensed matter physics**

Friday, November 30, 15:10 - 15:50 (MR3)

*Chair:* Joao Melo

The applications of the AdS/CFT correspondence to Physics are various and have reached plenty of different branches nowadays. In this presentation, a short review of the correspondence will be made, continuing with a brief introduction on the Condensed Matter phenomena of Quantum Criticality and Quantum Phase Transitions, illustrated with examples. Afterwards, some systems will be classified and described in terms of the

duality; and I will end with an insight into how to map temperature, chemical potential and magnetic fields for these systems via the correspondence.

### **3.6 Jude Pereira: Traversable wormholes**

Friday, November 30, 15:50 - 16:30 (MR3)

*Chair:* Joao Melo

After motivating the idea of traversable wormholes as a possible solution to the black hole information loss problem, we go on to investigate in detail the recent paper by Maldacena on 'Traversable Wormholes in four dimensions' which are stabilized by a peculiar motion of fermions in spacetime and hence, unlike its predecessors, does not demand the presence of exotic matter or concepts from AdS/CFT duality.

### **3.7 George Smith: Existence of a globally hyperbolic development**

Friday, November 30, 16:40 - 17:20 (MR3)

*Chair:* Jakub Supel

An existence theorem for Globaly Hyperbolic Developments of Cauchy data in General Relativity, using the nature of the Vacuum Einstein equations as a system of quasilinear wave eqautions.

### **3.8 Mauricio Doniz-Hernandez: Gravitational anomalies in black holes in string theory**

Friday, November 30, 17:20 - 18:00 (MR3)

*Chair:* Jakub Supel

Symmetries and their corresponding conservation laws play a fundamental role in the physical description of our universe. When a symmetry does not survive the transition from a classical to a quantum theory, we say it is anomalous. Anomalies arising from local symmetries lead to inconsistencies of the full quantum theory and therefore they must vanish. Robinson and Wilczek showed that after integrating out high frequency modes which cause a singular flux of the energy-momentum tensor near the horizon of a black hole, an effective (1+1)-dimensional anomalous theory sufficiently describes the physics near the horizon. The coefficients that cancel out the gravitational and gauge anomalies arising from this construction exactly coincide with the thermodynamical fluxes expected from a black hole, i.e. Hawking radiation. In this talk, I will revisit the R-W method and apply it to black hole solutions in the framework of string theory, originally obtained by Perry, Myers, et al. and succesfully recover the Hawking temperature of these black holes.

## 4 Statistical Physics

### 4.1 Zechen Zhang: A renormalization group approach to complexity

**CANCELLED**

Friday, November 30, 15:10 - 15:50 (MR14)

*Chair:* Fernando Caballero

Complex systems are often times hard to model completely and have novel emergent phenomena due to microscopic interactions. In this talk, I will focus on a particular case of complex network and examine how renormalization group can help us simplify the network without erasing characteristics that encode important information.

### 4.2 Anton Markov: Quantum Hall effect in an interacting model

Friday, November 30, 15:50 - 16:30 (MR14)

*Chair:* Fernando Caballero

A 2D lattice exposed to a strong uniform magnetic field shows intriguing physical phenomena. Its spectrum possesses a complex structure known as Hofstadter's butterfly featuring with the topologically protected quantized Hall Conductivity in the insulating phase. We will consider how this situation is affected by the Hubbard on-site interactions in the Dynamical Mean Field Theory approach.

### 4.3 Luke Corcoran: ODE/IM correspondence - quantising the quartic oscillator

Friday, November 30, 16:40 - 17:20 (MR14)

*Chair:* Fernando Caballero

Integrable systems in classical and quantum mechanics are those which can in some sense be solved exactly. In the quantum case there is no precise definition of integrability, although there are several features which arise time and again when solving systems which we would like to call 'integrable', such as the appearance of Bethe equations in finding the spectrum. In recent years an unexpected link between the spectral analysis of certain ODEs and quantum integrable models has been found, dubbed the 'ODE/IM correspondence'. In this talk I will discuss briefly the notion of integrability, and then demonstrate the simplest case of the correspondence which appears when quantising the quartic oscillator. I will show how there is a sense in which the quartic oscillator can be solved nonperturbatively, and back this up with some numerics.

#### 4.4 Gabriel Bliard: Kinetic field theory: a non-equilibrium statistical field theory describing cosmic structure formation

Friday, November 30, 17:20 - 18:00 (MR14)

*Chair:* Fernando Caballero

The perturbative methods in quantum field theory have applications far beyond calculating cross sections. Their use in statistical field theory is an example of how a physical concept can give rise to a transferable mathematical tool. Based on the work on kinetic field theory pioneered by Mazenko and Bartelmann, I will be describing these methods and presenting what is one of the first attempts to analytically model a many body gravitational system.

### 5 Quantum Field Theory and Symmetries

#### 5.1 Oleg Savchenko: Wigner's theorem and the Little group

Thursday, November 29, 16:40 - 17:20 (MR2)

*Chair:* Loren E. Held

Wigner's theorem tells us how the concept of symmetry is implemented in quantum mechanics: symmetries are realized by unitary or antiunitary operators. If one wants to understand properties of relativistic particles, one needs to find and classify all irreducible unitary representations of the Poincaré group. Wigner's classification does exactly that: starting from the most fundamental principles we know about nature, that is, relativity and quantum mechanics, it shows why particles are described by concepts such as mass and spin (or, in the massless case, helicity), and, in particular, it explains why 2 is not equal to 3, i.e., why massless particles have different number of degrees of freedom. In my talk, I am going to review the Wigner's theorem, concept of the little group and how it allows one to build all unitary irreps of the Poincaré group.

#### 5.2 Graham Van Goffrier: Kac-Moody algebras

Friday, November 30, 13:00 - 13:40 (MR2)

*Chair:* Laurence Cooper

Kac-Moody algebras are a superclass of Lie algebras as discussed in SFP this term, defined by relaxing the positive-definiteness requirement on the Cartan matrix.

### 5.3 Leonardo Badurina: The Bargmann superselection rule: understanding mass in non-relativistic quantum mechanics

Friday, November 30, 13:40 - 14:20 (MR2)

*Chair:* Laurence Cooper

A superselection rule (SSR) is commonly understood as expressing "restrictions on the nature and scope of possible measurements". Mathematically, this is equivalent to stating that the preparation of quantum states that exhibit coherence between eigenstates of certain observables is forbidden. In the context of non-relativistic (NR) quantum mechanics (QM), there exist superselection rules for the total mass of a system (i.e. Bargmann superselection rule), which imply that the superposition of two states, corresponding to different overall masses, does not define a pure state (i.e. these superpositions are forbidden). In this talk, after introducing the underlying theory of SSR, I will present how the Bargmann superselection rule arises as a consequence of demanding Galilean covariance of the NR Schrödinger equation. Finally, I will summarise recent dynamical implementations of the mass superselection rules in NRQM.

### 5.4 Yanbai Zhang: Kraft-Procesi transitions

Friday, November 30, 14:20 - 15:00 (MR2)

*Chair:* Laurence Cooper

By using the brane realisation of 3d  $N = 4$  gauge theory in type IIB supergravity, the minimal singularities can be identified and removed through Higgs mechanism in brane configurations. Such process is called the Kraft-Procesi transition.

## 6 Combinatorics

### 6.1 Huy Pham: Lower bounds for Szemerédi's regularity lemma

Friday, November 30, 13:00 - 13:40 (MR5)

*Chair:* Kasia Wyczesany

Szemerédi's regularity lemma is a powerful tool in extremal combinatorics giving a structural decomposition of all large graphs. However, its main drawback is the huge tower-type quantitative bound. We will attempt to sketch lower bound constructions showing that this bound is necessary, and if time permits, we will touch on other related aspects of the regularity lemma, including the arithmetic regularity lemma.

## 6.2 Wenqiang Xu: A robust version of Freiman's 3k-4 theorem

Friday, November 30, 13:40 - 14:20 (MR5)

*Chair:* Kasia Wyczesany

Freiman's 3k-4 Theorem tells us that if set  $A+A$  has size at most  $3|A| + 4$ , then  $A$  must "looks like" an arithmetic progression. We introduce a robust version (99% version) of the Freiman's 3k-4 Theorem and see some applications.

## 6.3 Maria Ivan: Colourings of the hypercube

Friday, November 30, 14:20 - 15:00 (MR5)

*Chair:* Kasia Wyczesany

We look at edge colourings of the cube with 2 colours and ask the question: is there always an antipodal geodesic that changes colours at most once?

# 7 Geometry and Algebra

## 7.1 Dan Kinch: Geometric algebra: complex numbers, quaternions, and beyond

Friday, November 30, 13:00 - 13:40 (MR4)

*Chair:* Adam Baranowski

Are there more general number systems with the flavour of the reals, complex numbers, quaternions, and octonions? Is there an analogue of the cross product that works in dimensions greater than three? Is there another way of thinking about rotations, reflections, and other geometric operations besides the usual matrix formalism? The tools maketh the mathematician, and we will answer these questions with the little-known tool of geometric algebra. After introducing some of the basic ideas of geometric algebra (such as bivectors and multivectors, or the exterior and geometric products), we will show how familiar faces like the complex numbers and the Pauli algebra arise naturally by applying these ideas to two- and three-dimensional spaces. We will then show how this machinery can be used to express rotations and reflections in a matrix-free algebraic way, discuss its connections to linear algebra and tensor analysis, and (time permitting) show how physical theories like relativity and quantum mechanics can be cast in the language of geometric algebra.

## 7.2 Dominic Wynter: Geometric measure theory: Plateau's problem

Friday, November 30, 13:40 - 14:20 (MR4)

*Chair:* Adam Baranowski

The Plateau problem asks for existence of a minimal surface with a given boundary, which was famously solved by Douglas and Rad in the 1930s, by reformulating the problem as a variational problem. Their proof brings together the theory of conformal maps, functional analysis, and convex analysis, via the Dirichlet energy. We give an exposition of this proof.

### **7.3 Michael Kopreski: A general basis for finitely supported G-equivariant maps**

Friday, November 30, 14:20 - 15:00 (MR4)

*Chair:* Adam Baranowski

Given a group  $G$  that acts on a set  $X$  and an integeral domain  $R$ , the contra/covariant action of  $G$  on  $X$ ,  $R$  induces an action on the  $R$ -module  $R[X]$  of finitely supported functions from  $X$  to  $R$ . If  $G$  acts trivially on  $R$ , then the characteristic functions of the orbits of  $X$  form a basis of the invariant functions of  $R[X]$ . In fact, even if  $R$  has a non-trivial  $G$ -action, one may construct a basis for the finitely supported  $G$ -equivariant maps using the orbits of an appropriately chosen subset of  $X$ . In this talk, I will prove the above claims; as examples, I will also demonstrate bases of the symmetric and antisymmetric polynomials.

## **8 Logic**

### **8.1 Joel Wee: A model-theoretic proof of the (weak) Nullstellensatz**

Friday, November 30, 13:40 - 14:20 (MR3)

*Chair:* Jose Vitor Paiva Miranda de Siqueira

Having built up our knowledge of Model Theory in the lecture course, we shall apply it to see a rather clean proof of Hilbert's Nullstellensatz.

### **8.2 Bhavik Mehta: Generalising induction, and coinduction**

Friday, November 30, 14:20 - 15:00 (MR3)

*Chair:* Jose Vitor Paiva Miranda de Siqueira

We explore the category of algebras for an endofunctor, and see how its initial object allows a generalised form of recursion and induction. Dually, we look at the category of coalgebras, and investigate corecursion and coinduction.

## 9 Number Theory

### 9.1 Soham Karwa: The shape of elliptic curves

Thursday, November 29, 14:20 - 15:00 (MR5)

*Chair:* Marius Leonhardt

Number Theory is hard. The field of Arithmetic Statistics arose from this frustration. In this talk, well be reviewing the recent work by Bhargava (for which he won his Fields medal) and Shankar. In particular, well be studying a particular invariant of elliptic curves, which formed the basis of my research project with Professor Thorne.

### 9.2 Evgeny Goncharov: Proof of the Weil conjectures

Thursday, November 29, 15:10 - 15:50 (MR5)

*Chair:* Marius Leonhardt

The Weil conjectures have been a driving force for the development of Algebraic geometry in the 1950-1970. I will state the Weil Conjectures, introduce the cohomological interpretation of the zeta-function and say a few words about Deligne's proof of the Riemann hypothesis for varieties over finite fields. Some knowledge of Algebraic geometry is necessary but I will try not to get too technical.

### 9.3 Jared Duker Lichtman: Primitive non-deficient numbers

Thursday, November 29, 15:50 - 16:30 (MR5)

*Chair:* Marius Leonhardt

We investigate the reciprocal sum of primitive nondeficient numbers, or pnds. Erdos showed that the reciprocal sum of pnds converges, which he used to prove that abundant numbers have a natural density. However no one has investigated the value of this series. We provide the first known bound by showing the reciprocal sum of pnds is between 0.348 and 0.380.

## 10 Quantum Computation, Information, and Foundations

### 10.1 Aleksandr Bowkis: Everettian Interpretations of quantum mechanics

Thursday, November 29, 15:10 - 15:50 (MR3)

*Chair:* Kirill Kalinin

Despite it's empirical success there remains much disagreement on the interpretation of quantum mechanics. This talk will discuss the modern formulation of Everettian style (many worlds) interpretations, focusing on the role of decoherence, emergence and decision theory.

## 10.2 Tien Sing Tay: Quantum game theory

Thursday, November 29, 15:50 - 16:30 (MR3)

*Chair:* Kirill Kalinin

In this talk, I aim to give an introduction to the subject area discussing the quantum formulation of game theory. We will then go on to examining the quantum versions of the Prisoners Dilemma and Newcombs game. Finally, we will discuss the interpretation of this in terms of real life negotiations. Not much previous knowledge is required besides elementary quantum mechanics.

## 10.3 Vu Phan Thanh: Markovian and time-homogeneous dynamical systems with quantum mechanics

Thursday, November 29, 16:40 - 17:20 (MR3)

*Chair:* Kirill Kalinin

Markov Chains and Markov Processes are basic stochastic time-evolutions in the classical theory of Probability. However: The natural foundation of this world - underpinned by results of physical experiments - is understood (and acknowledged) to be fundamentally not classical, but quantum. Firstly, this talk introduces the non-classical "probability distributions" in quantum mechanics (a.k.a. density operators or "quantum states"); then, it reviews classification results for feasible evolutions of quantum systems under the assumption of Markovian dynamics - in discrete time and in continuous time.

## 10.4 James Moore: Identical particles, path integrals, and quantum computers

Thursday, November 29, 17:20 - 18:00 (MR3)

*Chair:* Kirill Kalinin

We all know how exchange of identical particles in quantum mechanics gives rise to fermion and boson statistics. But what is really meant by 'identical particle exchange'? A proper treatment requires the path integral formulation of quantum mechanics, which leads to a surprising revelation - 2-dimensional particles don't have to be bosons or fermions! This talk will discuss the path integral formalism, 2D particles and their odd statistics,

and what these particles have to do with quantum computing. This talk will be accessible to anyone who knows basic quantum mechanics.